Tentamen 28.06.2011

№1

a) Massabalans voor stof A:

$$\frac{d}{dt}\left(V\cdot c_A\right) = \phi_V c_0 - \phi_V c_A$$

$$\frac{dc_A}{dt} = \frac{\phi_V}{V}\left(c_0 - c_A\right)$$

$$u = \frac{\phi_V}{V}\left(c_0 - c_A\right)$$

$$du = -\frac{\phi_V}{V}\, dc_A$$

$$-\frac{du}{dt}\cdot\frac{V}{\phi_V} = u$$

$$\frac{du}{u} = -\frac{\phi_V}{V}\, dt$$

$$\ln u = -\frac{\phi_V}{V} t + \ln A$$

$$u = A e^{-\frac{\phi_V}{V} t}$$

$$c_A = c_0 - \frac{A V}{\phi_V} e^{-\frac{\phi_V}{V} t}$$

$$c_A\Big|_{t=0} = 0 \Rightarrow A = \frac{\phi_V}{V} c_0$$

$$c_A = c_0\left(1 - e^{-\frac{\phi_V t}{V}}\right)$$

b) Met katalysator:

$$\frac{d}{dt} V c_A = \phi_V c_0 - \phi_V c_A - k_r \cdot c_A \cdot V$$

Als het toestand stationair is, $\frac{d}{dt} c_A = 0$

$$\phi_v C_0 - \phi_v C_A - k_r C_A \cdot V = 0$$

$$C_A = \frac{\phi_v \cdot C_0}{\phi_v + k_r V};$$

c) $$C_A = F(C_0, V, k_r, \phi_v, t)$$

6 variabelen => 6 - 3 = 3 dimensieloze groepen.

C_A : $\frac{kg}{m^3}$

C_0 : kg/m^3

V : m^3

k_r : s^{-1}

ϕ_v : m^3/s

t : s

$$\left(\frac{kg}{m^3}\right) = \left(\frac{kg}{m^3}\right)^{\alpha_1} \cdot \left(m^3\right)^{\alpha_2} \cdot \left(\frac{1}{s}\right)^{\alpha_3} \cdot \left(\frac{m^3}{s}\right)^{\alpha_4} \cdot \left(s\right)^{\alpha_5}$$

kg: $\alpha_1 = 1$

m: $3\alpha_1 - 3\alpha_2 - 3\alpha_4 = 3$

s: $\alpha_3 + \alpha_4 - \alpha_5 = 0$

α_4 en α_5 onafhankelijke variabelen.

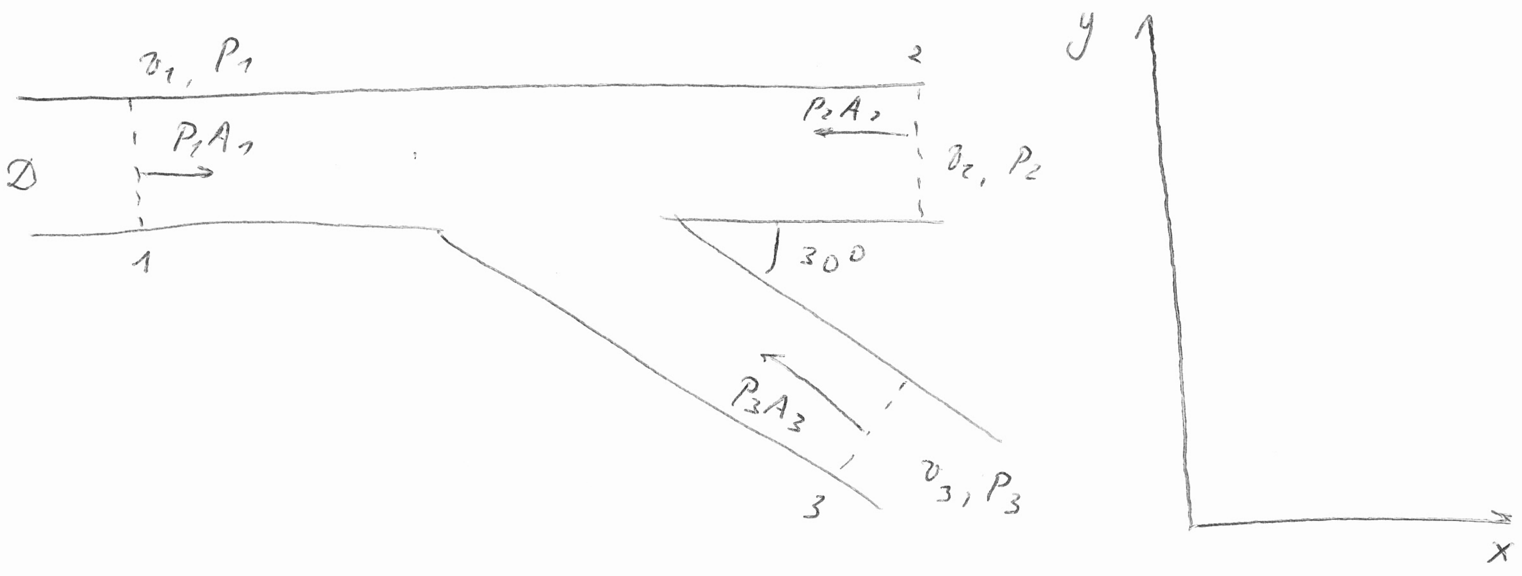
$\alpha_3 = \alpha_5 - \alpha_4$

$\alpha_2 = -\alpha_4$

$$C_A = C_0 \cdot V^{-\alpha_4} \cdot (k_r)^{\alpha_5 - \alpha_4} \cdot \phi_v^{\alpha_4} \cdot t^{\alpha_5}$$

$$C_A = C_0 \cdot \left(\frac{\phi_v}{V \cdot k_r}\right)^{\alpha_4} \cdot (k_r t)^{\alpha_5} \Rightarrow C_A = C_0 \cdot F\left(\frac{\phi_v}{V \cdot k_r}, k_r t\right)$$

N 2



1). Massa balans:

$$\rho_1 v_1 \cdot \frac{\pi D_1^2}{4} = \rho_2 v_2 \frac{\pi D_2^2}{4} + \rho_3 v_3 \frac{\pi D_3^2}{4}$$

$D_1 = D_2 = D_3$, $v_2 = v_3$, $v_1 = v$

$$v = 2 v_2 \Rightarrow v_2 = \frac{v}{2};$$

2). Bernoulie vergelijking:

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

$$P_2 = P_1 + \frac{\rho}{2}\left(v_1^2 - v_2^2\right) = P_1 + \frac{\rho}{2}\left(v^2 - \frac{v^2}{4}\right) = P_1 + \frac{3}{8}\rho v^2$$

$P_2 = P_3$;

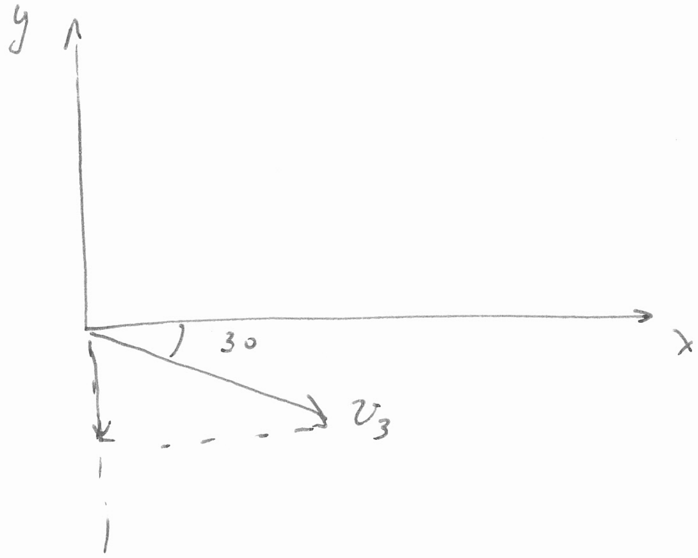
3) Impuls balans in de y-richting:

$$\frac{d}{dt}(M \cdot v_y) = \phi_{m,in} \cdot v_{y,in} - \phi_{m,uit} \, v_{y,uit} + \Sigma F_y$$

Het toestand is stationair $\Rightarrow \frac{d}{dt} M v_y = 0$

~~$\rho v_1 \cdot A_1 \cdot v_1 - \rho v_2 \cdot A_2 \cdot v_2 - \rho v_3 A_3 \cdot v_3 \cos 30^0 + \Sigma F_x = 0$~~

$v_{y,in} = 0$



$v_{y,uit} = -v_3 \cdot \sin 30^\circ$ (Negatief!)

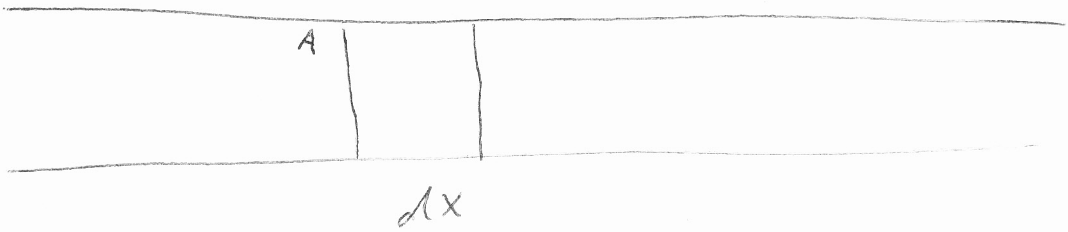
$-\rho v_3 A_3 \left(-v_3 \sin 30^\circ\right) + P_3 A_3 \sin 30^\circ + F_{w\text{-}v,y} = 0$

$F_{w\text{-}v,y} = \left[-\frac{\rho v^2}{8} - \left(p + \frac{3}{8}\rho v^2\right) \cdot \frac{1}{2}\right] \cdot \frac{\pi D^2}{4}$

$F_{w\text{-}v,y} = -\left[\frac{p}{2} + \frac{5}{16}\rho v^2\right] \frac{\pi D^2}{4}$

$F_{\cancel{y,w}\; v\text{-}w,y} = \left[\frac{p}{2} + \frac{5}{16}\rho v^2\right] \cdot \frac{\pi D^2}{4}$

N 3



Warmtebalans:

$$0 = \frac{\pi}{4} D^2 \rho v \, c_p \bar{T}\Big|_x - \frac{\pi}{4} D^2 \rho v c_p \bar{T}\Big|_{x+dx} + U \cdot \pi D dx (T_w - \bar{T})$$

$$0 = -\frac{D}{4} \rho v c_p \frac{d\bar{T}}{dx} + U dx (T_w - \bar{T})$$

$$y = T_w - \bar{T}$$

$$\frac{dy}{dx} = -\frac{4U}{D\rho v c_p} y$$

$$\frac{dy}{y} = -\frac{4U}{\rho v c_p D} dx$$

$$\ln y = -\frac{4U}{\rho v c_p D} x + \ln A$$

$$y = A e^{-\frac{4U}{\rho v c_p} x}$$

$$T_w - \bar{T} = A e^{-\frac{4U}{\rho v c_p} x}$$

$$\bar{T}\Big|_x = \bar{T}_0 \Rightarrow A = T_w - \bar{T}_0$$

$$\frac{T_w - \bar{T}}{T_w - T_0} = e^{-\frac{4U}{\rho v c_p D} x}$$

$$\bar{T} = T_w - (T_w - T_0) e^{-\frac{4U}{\rho v c_p D} x}$$

$$\cancel{\bar{T} = T_w = 250 - 230 e^{-\frac{\pi \cdot 0{,}2 \cdot 100}{10^3 \cdot 1} \cdot 10} = 127\,^\circ C}$$

$$X = \ln\left(\frac{T_w - \overline{T}_0}{T_w - \overline{T}}\right) \cdot \frac{\rho v c_p D}{4U}$$

$$\cancel{X = \ln \frac{250 - 20}{250 - 100}} \cdot$$

$$x = \ln\left(\frac{T_w - T_0}{T_w - \overline{T}}\right) \cdot \frac{\rho v \cdot \pi D^2}{4} \frac{c_p}{U \pi D}$$

$$X = \ln\left(\frac{T_w - T_0}{T_w - \overline{T}}\right) \cdot \phi_m \cdot \frac{c_p}{U \cdot \pi D}$$

$$x = \ln\left(\frac{250 - 20}{250 - 100}\right) \cdot 1 \frac{10^3}{100 \cdot \pi \cdot 0{,}2} = 6{,}8 \text{ m}$$

b) $$Re = \frac{\rho v D}{\mu} = \frac{4 \phi_m}{\mu \pi D} = \frac{4 \cdot 1}{2{,}4 \cdot 10^{-5} \cdot \pi \cdot 0{,}2} = 2{,}65 \cdot 10^5$$

=> turbulent.

$$Nu = 0{,}027\, Re^{0{,}8} \cdot Pr^{3}$$

$$Pr = \frac{c_p \mu}{\lambda} = \frac{10^3 \cdot 2{,}4 \cdot 10^{-5}}{0{,}025} \approx 1$$

$$Nu = 0{,}027 \cdot \left(2{,}65 \cdot 10^5\right)^{0{,}8} \cdot 1 \approx 590$$

c) $$x = \ln\left(\frac{T_w - T_0}{T_w - \overline{T}_1}\right) \frac{\rho v c_p D}{U_1}$$

$$x = \ln\left(\frac{T_w - T_0}{T_w - \overline{T}_2}\right) \frac{\rho v c_p D}{U_2}$$

$$\frac{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_1}\right)}{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_2}\right)} = \frac{U_1}{U_2}$$

$$U_2 = \frac{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_2}\right)}{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_1}\right)} U_1$$

$$\frac{1}{U_2} = \frac{1}{U_1} + \frac{1}{h_k} = \frac{1}{U_1} + \frac{d}{\lambda}$$

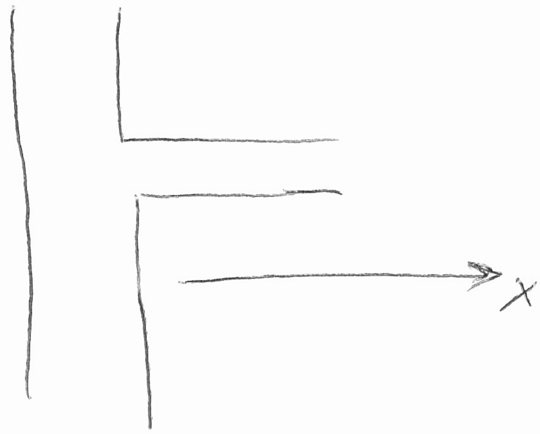
$$d = \lambda\left(\frac{1}{U_2} - \frac{1}{U_1}\right) = \lambda\left(\frac{U_1}{U_2} - 1\right)\frac{1}{U_1} =$$

$$= \frac{\lambda}{U_1} \cdot \left(\frac{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_1}\right)}{\ln\left(\frac{T_w - T_0}{T_w - \bar{T}_2}\right)} - 1\right)$$

$$d = \frac{0{,}03}{100}\left(\frac{\ln\frac{230}{150}}{\ln\frac{230}{170}} - 1\right) = 0{,}12 \text{ mm}$$

№4

a) Massa balans:



$$0 = -A\cdot D\frac{dC_B}{dx} + A(-v)\cdot C_B$$

$$\frac{dC_B}{dx} = -\frac{v\cdot C_B}{D}$$

$$\frac{dC_B}{C_B} = -\frac{v\,x}{D}$$

$$C_B = C_{B0}\,e^{-\frac{vx}{D}}$$

$$C_B < C_{B0}/100$$

$$1/100 > e^{-\frac{vx}{D}}$$

$$100 < e^{\frac{vx}{D}}$$

$$\frac{vx}{D} > \ln 100$$

$$v > \frac{D}{x}\ln 100$$

$$v > \frac{10^{-3}}{0.2}\ln 100 = 0{,}023\ \frac{m}{s}$$

b) Bernoullie vergelijking:

$$\frac{P_1}{\rho_1} + \frac{v^2}{2} = \frac{P_2}{\rho_2}\quad,\quad \rho_1 \approx \rho_2 \Rightarrow$$

$$P_2 = P_1 + \frac{\rho v^2}{2}$$

$$\Delta P = \frac{\rho v^2}{2}$$

$$\rho = \frac{P\cdot M}{RT} \Rightarrow \quad \Delta P = \frac{P\cdot M}{RT}\,\frac{v^2}{2}$$

$$\Delta P = \frac{28 \cdot 10^{-3} \cdot 10^{5}}{8{,}314 \cdot 293} \cdot \left(\frac{0{,}023}{2}\right)^2 = 3 \cdot 10^{-4}\ Pa$$

c). $\rho v \cdot A = \phi_m \cdot C$

waarin C massafractie van inert gas is.

$$C = \frac{\rho v A}{\phi_m} < C_L \qquad C_L = 10^{-5}$$

$$A < \frac{C_L \cdot \phi_M}{\rho \cdot v}$$

$$A < \frac{10^{-5} \cdot 1}{1{,}14 \cdot 0{,}023} \approx 0{,}00044\, m^2 = 4{,}4\ cm^2$$